

The Perils of Confounding Factors: How Fitts' Law Experiments can Lead to False Conclusions

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ABSTRACT

The design of Fitts' historical reciprocal tapping experiment gravely confounds index of difficulty ID with target distance D: Summary statistics for the candidate Fitts model and a competing model may appear identical, and the validity of Fitts' model for some tasks can be legitimately questioned. We show that the contamination of ID by either target distance D or width W is due to the common practices of pooling and averaging data belonging to different distance-width (D,W) pairs for the same ID, and taking a geometric progression for values of D and W. We analyze a case study of the validation of Fitts' law in eye-gaze movements, where an unfortunate experimental design has misled researchers into believing that eye-gaze movements are not ballistic. We then provide simple guidelines to prevent confounds: Practitioners should carefully design the experimental conditions of (D,W), fully distinguish data acquired for different conditions, and put less emphasis on r^2 scores. We also recommend investigating the use of stochastic sampling for D and W.

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INTRODUCTION

Fitts' law is a well known rule for human-aided movement that predicts the movement time (MT) it takes to reach a target of width W located at a distance D:

$$MT = a + b \log_2(1 + D/W) = a + b ID, \quad (1)$$

where ID is the index of difficulty¹ and a and b are estimated empirically.

Fitts' law gained importance in the HCI community after the seminal study by Card et al. [4] that measured the performance of four pointing devices on desktop computers. The law plays a prominent role in HCI, where it is heavily used to predict and evaluate the performance of input techniques. It has been shown to handle many conditions and tasks quite well, resulting in an incredibly wide spectrum of applications. Studies include reciprocal tapping between two targets using different limbs and body parts, such as tapping pedals [9] and manipulating a cursor [15] with the feet, controlling a stylus attached to the chin with head movements [2], and moving a cursor by rolling the head [16]. More unexpected applications include pointing and dragging sequences [12], rapid elbow flexion [6], eye-gaze movements [25, 21], and a study of patients with cerebral palsy [3],

Fitts' pointing paradigm for the reciprocal tapping experiment [10, Experiment I] has proven to be very influential. Fitts' apparatus was comprised of two plates of width W, separated by distance D. Distance and width were systematically varied: The four different conditions of D (2,4,8,16 (in.)) were crossed with four conditions for W (1/4,1/2,1,2 (in.)), resulting in 16 conditions. MT was then evaluated for each one of the 16 conditions. Today's version of a generic 1-D Fitts' law experiment is in many cases a simple adaptation of this protocol, where physical plates are replaced by targets on computer screens, and reciprocal tapping is sometimes replaced by discrete tapping, a cleaner version of the protocol due to Fitts & Peterson [11]. While the ISO standard [24] for the evaluation of pointing performance recommends a 2-D multi-directional tapping task, composed of circular targets of diameter W arranged in a circle of diameter D to control the effect of direction, many studies, old or recent, are conducted with the simple 1-D task.

Notations

Throughout the paper we use the following notations:

- *Factors* are noted in roman capital letters: distance D, width W, index of difficulty ID, and movement time MT.

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¹There are several formulations for ID, the one given here, known as the Shannon formulation [18, 19] being the most widely used in HCI.

- These factors take values in *sets*, denoted by their corresponding calligraphic capital letters: $\mathcal{D} = \{d_1, d_2, \dots, d_n\}$, $\mathcal{W} = \{w_1, w_2, \dots, w_m\}$, $\mathcal{ID} = \{id_1, id_2, \dots, id_k\}$.
- When two factors, e.g. D and W, are fully crossed, each element of \mathcal{D} is paired with each element of \mathcal{W} , resulting in $n \times m$ pairs. We use the condensed notation $\mathcal{D} \times \mathcal{W}$ to denote all these pairs.
- Several (D,W) pairs may result in the same D/W ratio. For example if $\mathcal{D} = \{10, 20, 50\}$ and $\mathcal{W} = \{1, 2, 4\}$, pairs (10,1) and (20,2) have the same D/W ratio of 10. We call \bar{D} the variable corresponding to the *average distance computed over equal ratios D/W*.
- The list of averaged values \bar{D} is noted $\bar{\mathcal{D}}$. With \mathcal{D} and \mathcal{W} as above, the set of ratios is $\{2.5, 5, 10, 12.5, 20, 25, 50\}$ and the corresponding list of values of \bar{D} is $\bar{\mathcal{D}} = (10, 15, 15, 50, 20, 50, 50)$.
- Similarly, we note \overline{MT} and \bar{W} the variables corresponding to the averages of MT and W computed over equal ratios D/W.

Flaws in Fitts' Literature

While Fitts' law has proven to be incredibly robust and useful, a number of potential flaws in the experimental designs used in the literature have been identified.

Guiard [14] showed that the design of Fitts' original tapping experiment correlates ID with D: "The dependence [between D and ID] is strong and systematic: on average target distance is raised by 11.7 cm for each extra bit of information" [14]. Hence Fitts' design makes D a *confounding* factor: a factor that has an effect on both a dependent and an independent variable in a controlled experiment.

It may then appear that manipulating the independent variable leads to variations in the dependent variable, when in fact both variations are due to the confounding factor. Thus, within Fitts' design, manipulating the ID affects movement time, but in fact both ID and MT are affected by the confounding factor D, making it impossible to disentangle the effects of D and ID on MT.

The correlations between D and ID are even stronger if one "builds an average on execution times for one ID first and calculate the correlation afterwards" [7]. This procedure, commonly used by HCI researchers, considers all blocks corresponding to the same ID as equivalent and computes a *single average per value of ID*, leading to what we have noted \bar{D} , \bar{W} and \overline{MT} . Not only does this operation pre-supposes the validity of Fitts' law [7, 8], it also strongly correlates \bar{D} and ID [14, 7, 8]. As we shall illustrate, it then becomes impossible to distinguish if the effects on \overline{MT} are due to \bar{D} or ID.

The implications of these confounds are wide ranging. According to MacKenzie [20], referring to Glencross and Barrett [13], "It has been suggested that the model [Fitts' law] would hold for the mouth or any other organ for which the necessary degrees of freedom exist and for which a suitable motor task could be devised". If D is indeed a confounding factor for ID, one can then wonder about the validity of Fitts' law in some of its applications.

Positioning and Goals of The Paper

One solution proposed by Guiard [14] to disentangle D and ID is what he called the complete form x scale design. Although this solution does guarantee the decorrelation of D and ID, it is impractical for pointing experiments in that it strongly restrains the range of variation of ID². The ID cannot be raised over 6 bits or so because that would require, at lower scale levels, impractically small values of W; nor can the ID be lowered below 4 bits or so because that would require, at higher scale levels, impractically large values of D [14].

While the form×scale design ensures a totally independent variation of factors D and ID, in "real-world" HCI, the decision about the validity of Fitts' law is usually based on the informal criterion that r^2 between ID and MT should be high enough (e.g., $r^2 > 0.9$ in [24]). What matters then, as we will show, is not that the confound be totally removed, but rather that its "strength" be reduced. We define X and Y as *strongly confounded* factors if $r^2(X, Y) > 0.9$, otherwise they are weakly confounded³. In contrast to Guiard [14], the focus of this paper is to find designs that prevent strong confounds.

The goal of this paper is threefold: First, we identify the objective conditions that lead to strong confounds in Fitts' law experimental designs. Such conditions can be evaluated on past and future experiments to quickly determine if the design is at risk of strong confounds. Second, once these conditions are clearly identified, we show a documented case in eye-gaze pointing where the validation of Fitts' model is the result of a strong confound between \bar{D} and ID. Third, we provide recommendations to protect the experimenter from confounding factors when designing experiments for Fitts' law. These recommendations may also prove useful for experimenters in HCI in general.

PRACTICAL EFFECTS OF STRONG CONFOUNDS

Guiard [14] and Drewes [7, 8] have illustrated the strong confound between \bar{D} and ID that occurs in Fitts' original design and raised ensuing qualitative issues. The goal of this section is to show a numerically worked out example of strong confounds between \bar{D} and ID, on a different design than Fitts'.

A Surprising Simulation

Let us consider two potential generative processes for MT:

Law A is Fitts' law, where MT is given by Eq. (1). Values for a and b are taken respectively as $a = -1047$ ms and $b = 391$ ms/bit.

Law B relates MT to D only:

$$MT = a' + b' D, \quad (2)$$

where $a' = -251$ ms and $b = 1.956$ ms/mm.

The explanation for the specific values of a , a' , b and b' will appear below. We now consider the following experimental

²As noted in [14], a full form×scale design on Fitts' design for Experiment I [10] requires, e.g., $W = 0.02$ cm.

³The operational value of r^2 above which a confound is deemed strong or not depends of course on what is expected by the experimenter. The value .9 is common in the Fitts' law literature.

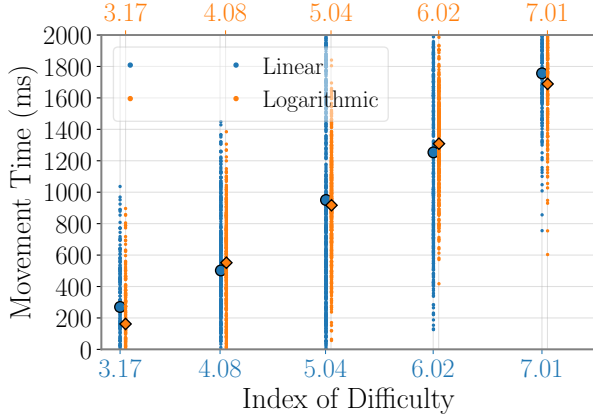


Figure 1. Simulated data for task A (orange, logarithmic) and task B (blue, linear). We have shifted the x-axis by 0.05 for task A for better legibility. The circles correspond to the average MT for a given ID for task B, the diamonds for task A. For Fitts' law (task A), we find $\overline{MT} = 386 \text{ ID} - 1040$ with $r^2 = 0.9995$. For task B, we find $\overline{MT} = 392 \text{ ID} - 1052$ with $r^2 = 0.9921$.

design: $\mathcal{D} = \{256, 512, 1024\}$ and $\mathcal{W} = \{8, 16, 32\}$, where \mathcal{D} and \mathcal{W} are fully crossed. There are thus 9 different experimental conditions that lead to only 5 different D/W ratios: (8,16,32,64,128). The corresponding average distances are $\overline{D} = (256, 384, 597, 768, 1024)$.

We run a simulation experiment where trials for each process are generated for the $\mathcal{D} \times \mathcal{W}$ conditions by adding a random noise sample to the underlying generative process. MT_i for each trial i is then given by,

$$MT_i = -1047 + 391 \log_2(1 + D/W) + z_i \quad (\text{law A}) \quad (3)$$

$$MT_i = -251 + 1.956 D + z'_i, \quad (\text{law B}) \quad (4)$$

where z_i and z'_i are the outcomes of a centered random process, which we took equally distributed according to a zero-mean Gaussian law with standard deviation $\sigma = 300$ ms, meaning that approximately 96% of the samples for MT are located within a 1200 ms interval. As we shall see later, the shape of the actual distribution is of little impact as long as it is centered.

We generated 200 trials per condition; the two datasets are represented in a MT vs. ID plane in Fig. 1. Five vertical scatters are obtained, one for each different D/W ratio. To get rid of the strong variability, we consider \overline{MT} , the averages of MT per ID. After averaging, we end up with the blue dots and orange diamond markers in Fig. 1.

The surprising result is that the summary \overline{MT} for law A is indistinguishable from that of law B. Both datasets, after averaging, are extremely well fitted with Fitts' law: The r-squared between \overline{MT} and ID is $r^2 = 0.9995$ for law A and $r^2 = 0.9921$ for law B. This is unexpected for the second dataset since it was generated with law B, not Fitts' law.

A Scenario for an Experimenter

The scenario for the above simulation is very close to a real experiment: Let us consider an experimenter who wants to investigate two tasks A and B and tries to find reasonable models to explain MT in both tasks. Let us assume that MT is actually governed by law A (Fitts' law) for task A and by law B for task B. The experimenter does not know this but has the *intuition* that *both* tasks should be reasonably well modeled by Fitts' law (law A).

After setting up the experimental design described above, he runs the experiment for both tasks. The resulting datasets are probably very similar to those in Fig. 1, with 5 vertical scatters for each task. After averaging over ID, he ends up with the large markers (blue disks and orange diamonds) of Fig. 1. This is precisely the averaging procedure discussed by Drewes [7] and mentioned in the introduction. After averaging, the experimenter then computes a fit using linear regression, and finds very high r^2 's for both tasks. He can thus conclude that his intuition was right and that Fitts' law is a good model to describe \overline{MT} for both tasks. Unfortunately, as we have shown, in the case of task B, this is an artifact due to the experimental design.

Decoding the Simulation

We now explain the results of the simulation.

Law A

For a given ID, the average of MT is equal to Fitts' law evaluated at this given ID, plus the average value of the z_i 's. Since the noise is centered, the law of large numbers implies that the average value of the noise is close to 0. While we used a Gaussian distribution, this holds for any shape of the noise distribution. We could also have used an asymmetric distribution to guarantee positive movement times, this would not have changed the values obtained after averaging. A similar observation can be made about the standard deviation. By virtue of the central limit theorem, a larger standard deviation would only require more trials for the sample mean to be close enough to the statistical mean.

Thus, the fact that we get an r^2 between \overline{MT} and ID close to 1 for law A is not surprising. As expected, we find that the linear regression between \overline{MT} and ID is very close to the one used for generating the data: $\overline{MT} = 386 \text{ ID} - 1040$ for the simulation results vs. $\overline{MT} = 391 \text{ ID} - 1047$ for the formula generating the data.

Law B

The dataset generated with law B has the same summary statistic as the one generated with law A. Although surprising, this can be explained as follows. In the MT versus ID plane, there can be more than one (D, W) condition leading to the same D/W ratio, e.g. for D/W = 32, there are three different values for D: D = 1024 (W = 32), D = 512 (W = 16), and D = 256 (W = 8). Therefore while it appears that there is only one vertical scatter plot per ID, there are in fact three overlapping ones. As the contribution of the noise will again be close to zero by the law of large numbers, average MT can thus be evaluated by inputting the average $D = 1/3 \times (256 + 512 + 1024) \approx 600$ into

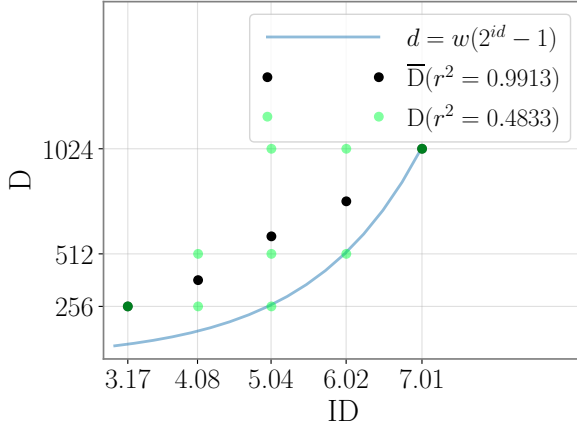


Figure 2. The 9 D conditions (light green dots) that lead to the 5 ID conditions used in the thought experiment. Black dots correspond to \bar{D} , the average value of D for equal IDs. Dark green dots correspond to data points that overlap the average. r^2 between D and ID and between \bar{D} and ID are given in the legend. The blue line corresponds to Eq. 8.

Eq. 2. By generalizing over all values of ID, \bar{MT} is obtained simply by evaluating Eq. 2 for each value of \bar{D} .

Fig. 2 plots the values of D appearing in the $\mathcal{D} \times \mathcal{W}$ conditions (light green dots) and those appearing in \bar{D} (black dots) against ID. The correlation between \bar{D} and ID is very high ($r^2 = 0.9913$). This means that the relationship between \bar{D} and ID can be considered linear:

$$\bar{D} = \alpha + \beta \text{ID}, \quad (5)$$

Linear regression gives $\alpha = -407$ and $\beta = 200$. Inputting Eq. (5) into Eq. (2), we find that \bar{MT} is given by

$$\bar{MT} = a' + b' \bar{D} = a' + b' \alpha + b' \beta \text{ID} = a'' + b'' \text{ID}, \quad (6)$$

thereby showing that \bar{MT} will indeed appear linear in ID. Notice that this situation can only happen if the confound between ID by \bar{D} is strong enough, which warrants our focus on strong confounds.

The values used in the simulation can now be explained: We chose the values of a and b and computed a' and b' from Eq. (6). We can verify that $a = a' + b' \alpha = -251 + 1.956 \times -407 = -1047$ and $b = b' \beta = 1.956 \times 200 = 391$.

Strong Confounds in the Goal Passing Task

The previous simulation is not entirely artificial. In fact, the $\mathcal{D} \times \mathcal{W}$ conditions, as well as the values⁴ of a and b are from Accot & Zhai’s goal passing task [1, Experiment 1]. They conducted this experiment to validate Fitts’ law as a model for

⁴Accot & Zhai [1] actually give $a = -1347$ instead of the -1047 that we used. We did so because the values of the fit given in their paper do not match the plot in their figure. For example, for $\text{ID} = 3.5$, we can read $\text{MT} \approx 300$ ms in the graph, but a calculation using the values of the fit predicts 21.5 ms. We assume that there was a typographical error, whereby the “0” in 1047 was mistakenly typed as a “3”.

goal passing, a result that they used in the derivation of the steering law.

Accot & Zhai used 9 conditions, yet only represented 5 movement time averages [1, Fig. 3], each corresponding to a different ID, meaning that they considered \bar{MT} . According to the simulation above, we now know that another law than Fitts’ law, whose formula is given by Eq. (2), will fit \bar{MT} equally well. Note that since one law depends on W and not the other, one of the two models must prove inaccurate in a design that does not confound \bar{D} and ID so strongly.

Accot & Zhai were apparently unaware of this difficulty, and concluded that the “goal passing task follows the same law as in Fitts’ tapping task despite the different nature of movement constraint”. They used the word “despite” as if surprised that Fitts’ law proved a good predictor for movement times.

In fact, the law given by Eq. (2) seems reasonable for a goal passing task when W is large enough, since in that case the width does not really constrain the movement and the goal passing task becomes a simple distance covering task. Assuming a constant maximum speed c , movement time would simply be given by

$$\text{MT} = t_0 + 1/c \times (D - d_0), \quad (7)$$

where t_0 is the time needed to reach a speed of c , and d_0 the distance traveled until c is reached. This is a linear model as in Eq. (2) with slope $1/c$ and intercept $t_0 - d_0/c$.

CONDITIONS FOR STRONG CONFOUNDS

Fitts’ design for the tapping experiment [10, Experiment I] strongly confounds \bar{D} with ID (r^2 between \bar{D} and ID above .99, see [14, 8]). We have established that the design of the goal passing task [1, Experiment 1] suffers from a similar confound (r^2 between \bar{D} and ID above .99). In this section, we investigate the reason for these strong confounds.

Fitts-Like Designs

We first define a class of experimental designs, which we call *Fitts-like designs*, characterized by experimental conditions of the following general form:

1. $\mathcal{D} = \{d \cdot 2^i\}$, where $0 \leq i \leq N - 1$ and d is fixed,
2. $\mathcal{W} = \{w \cdot 2^j\}$, where $0 \leq j \leq M - 1$ and w is fixed.
3. \mathcal{D} and \mathcal{W} are fully crossed.

Conditions 1 and 2 state that the values of D and W follow a geometric progression. Both Accot & Zhai’s goal passing task and Fitts’ tapping experiment are Fitts-like designs.

Table 1 shows the r^2 values between \bar{D} and ID for four Fitts-like designs: “Tapping”, “Disc Transfer” and “Pin Transfer” refer to the experiments conducted by Fitts [10]; “Goal Passing” refers to the study conducted by Accot & Zhai [1, Experiment 1]. All four experiments lead to strong confounds between \bar{D} and ID.

We also compute the r^2 values between \bar{D} and ID for different sizes of \mathcal{D} and \mathcal{W} . Fig. 3 shows that any Fitts-like design strongly confounds \bar{D} with ID if N and M are small enough. Practical considerations often limit the values of N and M to

XP	N/M	d	w	$r^2(\bar{D}, ID)$	$r^2(\bar{W}, ID)$
Tapping	4/4	2(in.)	1/4(in.)	0.99	0.94
Disc Transfer	4/4	4(in.)	1/16(in.)	0.97	0.97
Pin Transfer	5/4	1(in.)	1/16(in.)	0.95	0.94
Goal Passing	3/3	256(pix.)	8(pix.)	0.99	0.99

Table 1. Characteristics of four Fitts-like experiments. Tapping, Disc Transfer and Pin Transfer by Fitts (1954) and Goal Passing by Accot & Zhai (1997).

about 5, so that most Fitts-like designs are likely to produce strong confounds between \bar{D} and ID.

Geometric Progression of D&W Causes Strong Confound

The strong confound between \bar{D} and ID is the result of the geometric progressions of D and W. First, notice that D is an exponential function of ID (Fig. 2):

$$d = w \times (2^{id} - 1) \quad (8)$$

For \bar{D} to be strongly confounded with ID, i.e. to be linearly dependent of ID, \bar{D} must combine several values corresponding to the same ID. If a design is fully crossed, there are at least two values of ID corresponding to a single (D, W) condition, namely the minimum ID (minimum D associated with maximum W) and the maximum ID (maximum D associated with minimum W). In the case of Fitts-like designs, only these two IDs meet this condition, as can be seen in Fig. 2.

Let us now construct a design where there are multiple values of D for each ID, except for the extreme ones. We start with a predetermined set $ID = \{id_1, id_2, \dots\}$ of increasing IDs. We can then always choose the values of D and compute the corresponding W using the definition of ID. Assuming that the design is fully crossed, the smallest ID, id_1 , is necessarily composed by the smallest D (d_1) and the largest W (w_1). We pick an arbitrary value for d_1 and solve $id_1 = \log_2(1 + d_1/w_1)$ for w_1 , giving

$$w_1 = d_1 / (2^{id_1} - 1)$$

Since we do not allow a single D condition for a given value of ID (except at the edges), the second smallest value of ID, id_2 , should correspond to the two combinations $d_1 \times w_2$ and $d_2 \times w_1$, from which the values of w_2 and d_2 can be computed:

$$id_2 = \log_2(1 + d_1/w_2) \implies w_2 = d_1 / (2^{id_2} - 1)$$

$$id_2 = \log_2(1 + d_2/w_1) \implies d_2 = w_1 \times (2^{id_2} - 1)$$

Note that we have $d_2/d_1 = w_1/w_2$.

The next smallest value of ID, id_3 , must correspond to at least two of the following combinations: (d_3, w_1) , (d_1, w_3) , (d_2, w_2) , i.e. $d_3/w_1 = d_2/w_2$ and $d_1/w_3 = d_2/w_2$. Solving these gives $d_3 = d_2 w_1 / w_2$ and $w_3 = w_2 d_1 / d_2$. Note that we have $d_3/d_2 = w_1/w_2 = d_2/d_1$ and $w_3/w_2 = d_1/d_2 = w_2/w_1$.

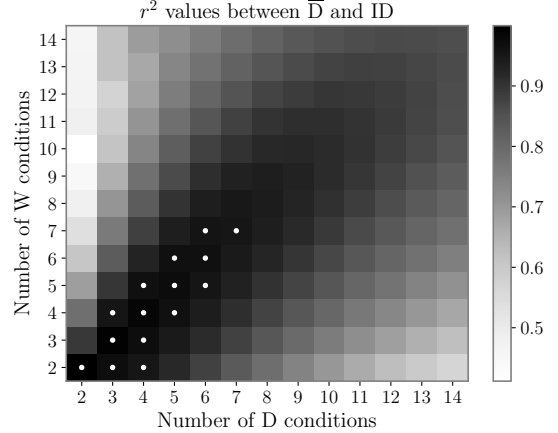


Figure 3. r^2 values between \bar{D} and ID when N (number of D conditions) and M (number of W conditions) are varied. White dots indicate the cases where $r^2 \geq 0.95$. r^2 computed for $d = 128$ and $w = 12$ according to our notation of Fitts-like designs.

We repeat this procedure for the remaining values of ID. It follows that the increasing sequence of distances d_i and the decreasing sequence of widths w_j are such that all ratios d_i/d_{i+1} and w_{j+1}/w_j are equal, except at the edges, resulting in geometric progressions for the values of D and W.

The fact that the resulting values of \bar{D} are almost in linear progression can be explained mathematically, but is of little interest: it is a coincidence due to the fact that the range of IDs being investigated is small. This explains why r^2 decreases when N and M increase, as shown in Fig. 3.

To summarize, the fact that Fitts-like designs create strong confounds between \bar{D} and ID can be attributed to the following causes:

1. D and W are both in geometric progressions (one passes from one value to the next by multiplying by some constant, e.g. 2);
2. the range of IDs used in practice is small.

Other Strong Confounds

We have just identified the reasons that make Fitts-like designs strongly confound \bar{D} with ID. As W grows at the same rate as D, one would expect an equivalently strong confound in Fitts-like designs between \bar{W} and ID. This is indeed the case as can be seen in the rightmost column of Table 1. One should thus also be careful to avoid strong confounds between \bar{W} and ID.

Care must also be taken with designs that are not Fitts-like. The combination of a small number of conditions, the fact that they are usually chosen according to some structure (such as a linear or geometric progression), and the smoothing effect of averaging make it very likely that \bar{D} or \bar{W} can be approached by a simple function of ID.

We illustrate this with an example. Consider a candidate model⁵ for the dependent variable Y as a function of the independent variable X :

$$Y = f(X). \quad (9)$$

Now consider a third variable Z , that can be expressed as a function of X :

$$Z = g(X). \quad (10)$$

We will call g the *confusion* function. A competing model of the form

$$Y = f(g^{-1}(Z)) \quad (11)$$

will inevitably be indistinguishable from the candidate model, since plugging (10) into (11) gives:

$$Y = f(g^{-1}(g(X))) = f(X) \quad (12)$$

which is equivalent to Eq. (9).

The variable Z used here is very general and can represent any factor. In the example developed in the previous section (“Thought experiment”), Z was \bar{D} and the g was a linear function. As another, more complex example, let us consider two sets \mathcal{D} and \mathcal{W} for which \bar{D} has a square root relationship to ID

$$\bar{D} = a' \sqrt{\text{ID} + b'} \quad (13)$$

Then, $g^{-1}(x) = (x/a')^2 - b'$, and $f(g^{-1}(x)) = a - bb' + b(x/a')^2$, so that a quadratic model for MT cannot be distinguished from Fitts’ model, as MT is also linearly related to ID. If we then take a model of the form

$$\overline{\text{MT}} = a - bb' + b(\bar{D}/a')^2 \quad (14)$$

plugging Eq. (13) into Eq. (14) gives

$$\overline{\text{MT}} = a + b\text{ID}. \quad (15)$$

This is a simple linear function between $\overline{\text{MT}}$ and ID, thereby showing that a quadratic law for movement time can be indistinguishable from Fitts’ law in some designs.

While in general, a weak confound between \mathcal{D} and ID implies that any function of \mathcal{D} is (weakly) confounded with ID, this is not true anymore for strong confounds. We have shown here that most designs of Fitts’ law experiments are likely to have a strong confound between ID and *some function* of \bar{D} and \bar{W} . If one knows precisely this strong confound, e.g. Eq. (13), it is easy to determine which model for MT will have almost the same summary statistic $\overline{\text{MT}}$ and r^2 as Fitts’ model. For example, with Eq. (13), consider the model of Eq. (14).

Creating Strong Confounds Between Any Two Factors

In the previous subsections, we explained how strong confounds between ID and functions of \mathcal{D} or \mathcal{W} could make two models indistinguishable from each other. We now give a general method to create strong confounds between ID and almost any function of \mathcal{D} or \mathcal{W} , as a constructive illustration.

⁵In Fitts’ law, $Y \equiv \text{MT}$, $X \equiv \text{ID}$, and f is a linear function. X , Y and Z may also represent averaged quantities, e.g. \bar{D} or $\overline{\text{MT}}$.

1. Choose a set of ID values \mathcal{ID} ,
2. Choose a set of target sizes \mathcal{W} .
3. Choose the confusion function, e.g. $\bar{D} = 4.5 \sqrt{\text{ID} - 0.9}$.
4. For each target size $w \in \mathcal{W}$ and for each $\text{id} \in \mathcal{ID}$, find the corresponding $D_{w,\text{id}} = (2^{\text{id}} - 1)w$.
5. For each ID, find the combination of \mathcal{D} ’s whose average minimizes the distance to the target confusion function.

Figure 4. A generic method that generates a design to create confounding variables

Fig. 4 shows a method for creating $(\mathcal{D}, \mathcal{W})$ pairs that strongly correlate ID with any function of \bar{D} . Confounding with \bar{W} is easily achieved by switching the roles of \mathcal{D} and \mathcal{W} . Extra search steps could be added to the algorithm. For example, in step 2, one could consider several sets \mathcal{W} and keep the one that minimizes the distance to the target confusion function; in step 3, the parameters of the confusion function could be varied.

We used the procedure described in Fig. 4 with the following confusion function from Eq. (13):

$$\bar{D} = 4.5 \sqrt{\text{ID} - 0.9}. \quad (16)$$

The corresponding \mathcal{D} and \mathcal{W} conditions, resulting in 12 pairs, are given in Fig. 5. Notice that in resulting design \mathcal{D} and \mathcal{W} are almost as well decorrelated as in a fully crossed design.

We then perform the same simulation as in the Thought Experiment: Law A is given by Fitts’ law, as before, and Law B is the quadratic law in Eq. (14). After the simulation of 200 trials per condition, we find that r^2 between $\overline{\text{MT}}$ and ID is once again very high for Law A: $r^2 = 0.99$. It is also very high for Law B: $r^2 = 0.95$. As in the Thought Experiment, the summary statistics of the two laws are almost identical. More importantly, we obtain very good fits for movement time using Fitts’ law in a situation where the data for movement time was generated using a quadratic law that does not depend on the width factor \mathcal{W} .

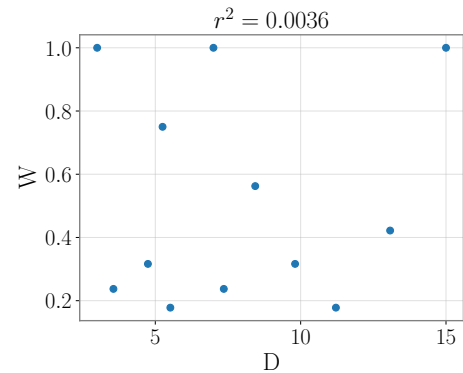


Figure 5. \mathcal{D} and \mathcal{W} conditions that lead to an almost perfect confusion between \bar{D} and $\sqrt{\text{ID}}$. The correlation between \mathcal{W} and \mathcal{D} is $r^2 = 0.0036$.

In summary, we first demonstrated that Fitts-like designs create strong confounds between \bar{D} and ID. We then showed that the issue is much deeper than Fitts-Like designs: With any design, there exists a possibility that some function of D or W is strongly confounded with ID.

EYE-GAZE EXPERIMENTS

We discuss two studies conducted on eye-gaze movements using the results of the previous section. These demonstrate that the issues we have underlined until now are not only theoretical constructs, but do appear in real-world scenarios. The first study, by Miniotos [21], validated Fitts' model for movement time in a pointing task using eye-gaze. Drewes [8] discussed this study and pointed out that outside HCI, researchers would generally use Carpenter's formula [5] (a formula not dependent on W) to model eye-gaze data. We compare this first study to another study by Miniotos et al. [22], as yet uncommented, and show that the results of these two studies on the validity of Fitts' model for eye-gaze data are strikingly inconsistent. We will explain the difference by formally applying Carpenter's formula to Miniotos' paradigm and using the previous results on Fitts-like designs.

Fitts' Law for Eye-Gaze Interaction

The first study [21] was conducted to validate Fitts' law for modeling eye-gaze interactions. According to Miniotos [21], prior work by Ware and Mikelian [25] suggested that Fitts' law would be an adequate model for movement time in eye-gaze interaction, but the range of ID explored then was very narrow (less than 3 bit wide), and the width W had been kept constant. The motivation was thus to create an empirical dataset that was more thorough.

In line with Fitts' paradigm, the task was to move the cursor to a target of width W located at a distance D by using their eyes instead of a stylus. An eye tracker was used to control the cursor. The control variables were D ($D = \{26, 52, 104, 208\}$ (mm)) and W ($W = \{13, 26\}$ (mm)). It is not clear how the r^2 between MT and ID was computed, but Miniotos reported $r^2 = 0.982$. He concluded that Fitts' law was a good fit for his dataset, a valuable result for designers.

However, it turns out that the design of the experiment is "Fitts-like" according to our definition: First, both D and W have a geometric progression; there are 8 conditions, yet only 5 different D/W ratios. Second, the range of IDs, $[1, 4.1]$, is quite small. Accordingly, the correlation between \bar{D} and ID is very strong: $r^2 = 0.974$, leading to a strong confound. As a consequence, a competing model such as Eq. (2) could equally well explain the summary of the gathered data. Note that Ware & Mikelian [25], recognizing that eye-saccades are ballistic, stated that they had used Fitts' law "only as a convenient way of summarizing the results, not because [they] wish[ed] to make any theoretical claims".

Carpenter's Formula

A reliable relation known as Carpenter's formula [5] relates MT and angular amplitude α (Eq. (17)) for eye-gaze movements:

$$MT = a + b \alpha, \quad (17)$$

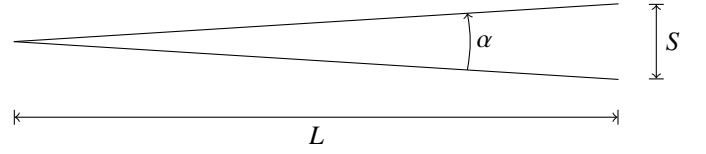


Figure 6. Carpenter's Formula applied to Fitts' pointing paradigm. L is the fixed distance between the user and the screen, S is the on-screen distance that the cursor must travel, i.e. D with the notations of Eq. (1).

where MT is the movement time needed to cover the angular amplitude α . Carpenter's formula can be applied to Miniotos' paradigm as illustrated in Fig.6, where L is fixed and S corresponds to D in Fitts' paradigm. In line with the ballistic nature of eye movements, W does not appear in Carpenter's formula. The angle α can be expressed in terms of the available parameters as

$$\alpha = 2 \arctan \left(\frac{S}{2L} \right) \approx \frac{S}{L} = \frac{D}{L} \text{ when } \alpha \text{ is small enough.} \quad (18)$$

If we input this equation into formula Eq. (17), we obtain

$$MT = a + b \alpha \approx a + b' D, \text{ where } b' = b/L. \quad (19)$$

Therefore, Carpenter's formula, the leading explanatory model, predicts that movement time is linearly related to D .

Thanks to our previous analysis, we can now safely assert that since the design of Miniotos' experiment [21] is Fitts' like, Fitts' model is almost equivalent in terms of fitting \bar{MT} to the leading explanatory model derived from Carpenter's formula. Therefore one cannot conclude from that experiment that eye-gaze follows Fitts' law.

Eye-Gaze Interaction with Expanding Targets

Miniotos et al. [22] conducted a second study on eye-gaze interaction with expanding static targets. Expanding static targets are targets whose appearance does not change for the user, but to which the interface responds as if it were larger. The expansion is predetermined, hence the term "static".

Although the goal was not to validate Fitts' law, Miniotos et al. did check for goodness of fit of the Fitts model and found $r^2 = 0.69$. The experiment is very similar to that described in the first study [21], with a notable exception: the progression for W is not geometric, but linear: $D = \{128, 256, 512\}$ (pix.) and $W = \{12, 24, 36\}$ (pix.). Because this design is not Fitts-like, we expect r^2 between \bar{D} and ID to be weaker than for Fitts-like designs, and indeed we find $r^2 = 0.74$.

Miniotos et al. attributed the decrease in correlation between MT and ID from 0.98 in the 2000 study [21] to 0.69 in the 2004 study [22] to the presence of a visible cursor in the 2000 study,

XP	Fitts-like	$r^2(\bar{D}, id)$	$r^2(MT, id)$
Miniotos 2000	yes	0.99	0.98
Miniotos et al. 2004	no	0.74	0.69

Table 2. Table summarizing the relevant r^2 in the Miniotos (2000) and Miniotos et al. (2004) studies. $r^2(x, y)$ is the coefficient of determination between x and y .

whereas there was no visual feedback in the 2004 study. This is because they assumed Fitts' law to be a valid model. However, the comparison between the two studies tells a different story, as shown in Table 2. The design used in the 2000 study [21] was Fitts-like, making Fitts' model almost equivalent to the one derived from Carpenter's formula, whereas the design used in the 2004 study [22] was not Fitts-like. As a result the confound between \bar{D} and ID is not as strong (r^2 between \bar{D} and ID of .74). Not surprisingly, r^2 between \bar{MT} and ID drops with a similar magnitude (r^2 between \bar{MT} and ID of .69). We conclude that Fitts' model's apparent validity in [21] is an artifact caused by the experimental conditions.

COMBATING STRONG CONFOUNDS

In this section we give four recommendations to protect experiment designers from strong confounds.

Do Not Trust a Good r^2

The two datasets generated from law A and law B in the Thought Experiment had the same \bar{MT} and r^2 . From this summary alone, they were indistinguishable. If one had used different summaries, such as those from Jude et al. [17], striking differences would have emerged. For example, the variance of the two datasets are very different.

The evaluation of Fitts' model in the HCI community relies almost exclusively on high r^2 values. As emphasized by Roberts & Pashler [23] however, a good fit reveals nothing about the flexibility and variability of the data (what the model can and cannot fit), nor the likelihood of other models. Indeed, we have shown that within a Fitts-like design, two different models could fit the same summary data.

Do Not Average or Pool Data From Different Conditions

We have seen (Sect. 3) that the strong confound between \bar{D} and ID was made possible because of the averaging procedure. For example, in Fitts' tapping experiment, the r^2 between D and ID is .48; after averaging the r^2 between \bar{D} and ID is .99. It is unfortunately common for experimenters to pool data and average movement times that correspond to the same D/W ratio but that do not come from the same $D \times W$ condition (see, e.g., Drewes [7, 8]). This practice involves a confirmation bias: The logic behind averaging movement times that correspond to the same D/W ratio but come from different pairs (D,W) is to believe that because data was acquired under the same ratio and that movement time is supposedly dependent on the ratio only, the two conditions would essentially be the same. But this is only true if indeed, the ratio explains all the variability of MT, which is precisely what we want to test when trying to validate Fitts' law in the first place.

Averaging before knowing the validity of Fitts' model may then result in a premonitory experiment where Fitts' law can be validated simply because it was pre-supposed to hold, as shown in the section analyzing Miniotas' experiment [21].

Note that for experimenters using the effective index of difficulty, ID_e [24], a different value of ID_e is calculated for each block based on the participants variability in that specific block. Therefore different conditions will almost always result

in different values of ID_e , even if they correspond to the same D/W ratio. The net result is that this procedure eliminates the risk of averaging across (D, W) conditions.

Consider Competing Models

We have shown that Fitts-like designs create strong confounds between ID and both \bar{D} and \bar{W} . We further showed that we could construct a design that strongly confounds ID with almost any simple function of D or W. It is then important, when evaluating whether Fitts' model is a good fit for a task, to also consider competing models. If there are any, the experimenter should make sure that the design does not strongly confound factors of both models. For example, in the eye-gaze study it would have been safer to also evaluate Carpenter's formula (Eq. 17) on the experimental data.

Notice that once a competing model is identified, it is easy to verify the risk of strong confounds among factors by checking the correlations between them.

Use Stochastic Conditions

In a Fitts' law experiment, D and W are varied and MT is measured. The average MT of each block represents one sample in the (D,W) space. Experimental data can thus be visualized as a set of samples in the (D,W) space⁶. Fitts [10] showed that this representation could be summarized by a simple formula – now known as Fitts' law. We have shown that some sampling strategies such as Fitts', i.e. geometric progressions and orthogonal sampling in the (D,W) space, may lead to strong confounds between factors. A different sampling strategy is Guiard's [14] orthogonal sampling in the form×scale space, which provides a theoretical solution to the issue, but can lead to physical values of D and W that are hard to implement in practice.

We have shown that sampling issues leading to strong confounds occur under very specific conditions, i.e. when the conditions are generated by some rule. For example, a Fitts-like design is characterized by a geometric progression for D and W. Therefore, we believe that a simple solution to get rid of potential sampling artifacts is to adopt stochastic conditions for D and W, possibly with some constraints. For example, we could divide the (D,W) space into a grid and choose experimental conditions by drawing a point uniformly within each rectangle defined by the grid, resulting in pairs of (D,W) values).

CONCLUSION AND PERSPECTIVES

In the experimental testing of any mathematical model, we may distinguish two steps:

1. Sampling the factor space, e.g. Fitts' traditional (D, W) space or Guiard's form×scale space, thus defining a set of experimental conditions for data collection;
2. Processing the data by applying operations that yield a score. In Fitts' law studies, this traditionally involves computing means of movement times and r^2 values between ID and \bar{MT} .

⁶Incidentally, this is precisely how Fitts summarized his data in his historical study [10, Fig. 4]

We have shown that a Fitts-like sampling of the (D, W) space solely associated with the computation of r^2 between ID and MT creates strong confounds between \bar{D} and ID. We attributed this to the geometric progression of D and W. A simple workaround would seem to be to avoid such a sampling. However, using a constructive approach, we devised a sampling strategy that strongly confounds ID with any simple function of D and W. Avoiding Fitts-like designs is thus insufficient to avoid strong confounds.

Based on these new results, we analyzed an apparent contradiction between the results of two eye-gaze pointing experiments using. We resolved the contradiction by noting that Carpenter's formula is a widely accepted model for eye-gaze data and by showing that in one of the experiments, Fitts' model was indistinguishable from Carpenter's model due to the use of a Fitts-like design.

Finally, we provide guidelines to avoid strong confounds between factors. We believe that Fitts' law studies place too much emphasis on high r^2 values. It is crucial to introduce other considerations when validating a model, such as the flexibility of the evaluated model, the variability of the dataset and the possibility of competing models. Working with block averages or, worse, averages computed for equal values of ID, such as MT, dramatically decreases the number of points to be fitted, thereby mechanically increasing r^2 values.

An interesting and simple way to prevent strong confounds is to use stochastic sampling of the (D,W) space. Stochastic sampling is a promising perspective, especially when considering the replication of studies, as a different but equivalent design can be ensured with each replication. However, more conceptual work is needed to support the idea of using random conditions in a controlled experiment.

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